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Motion in One Dimension

CHAPTER OUTLINE

- 2.1 Position, Velocity, and Speed
- 2.2 Instantaneous Velocity and Speed
- 2.3 Acceleration
- 2.4 Motion Diagrams
- 2.5 One-Dimensional Motion with Constant Acceleration
- 2.6 Freely Falling Objects
- 2.7 Kinematic Equations Derived from Calculus

ANSWERS TO QUESTIONS

- Q2.1** If I count 5.0 s between lightning and thunder, the sound has traveled $(331 \text{ m/s})(5.0 \text{ s}) = 1.7 \text{ km}$. The transit time for the light is smaller by

$$\frac{3.00 \times 10^8 \text{ m/s}}{331 \text{ m/s}} = 9.06 \times 10^5 \text{ times,}$$

so it is negligible in comparison.

- Q2.2** Yes. Yes, if the particle winds up in the $+x$ region at the end.

- Q2.3** Zero.

- Q2.4** Yes. Yes.

- Q2.5** No. Consider a sprinter running a straight-line race. His average velocity would simply be the length of the race divided by the time it took for him to complete the race. If he stops along the way to tie his shoe, then his instantaneous velocity at that point would be zero.

- Q2.6** We assume the object moves along a straight line. If its average velocity is zero, then the displacement must be zero over the time interval, according to Equation 2.2. The object might be stationary throughout the interval. If it is moving to the right at first, it must later move to the left to return to its starting point. Its velocity must be zero as it turns around. The graph of the motion shown to the right represents such motion, as the initial and final positions are the same. In an x vs. t graph, the instantaneous velocity at any time t is the slope of the curve at that point. At t_0 in the graph, the slope of the curve is zero, and thus the instantaneous velocity at that time is also zero.

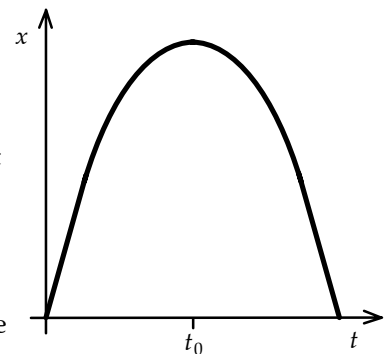


FIG. Q2.6

- Q2.7** Yes. If the velocity of the particle is nonzero, the particle is in motion. If the acceleration is zero, the velocity of the particle is unchanging, or is a constant.

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Q2.8 Yes. If you drop a doughnut from rest ($v = 0$), then its acceleration is not zero. A common misconception is that immediately after the doughnut is released, both the velocity and acceleration are zero. If the acceleration were zero, then the velocity would not change, leaving the doughnut floating at rest in mid-air.

Q2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past.

Q2.10 Yes. Consider throwing a ball straight up. As the ball goes up, its velocity is upward ($v > 0$), and its acceleration is directed down ($a < 0$). A graph of v vs. t for this situation would look like the figure to the right. The acceleration is the slope of a v vs. t graph, and is always negative in this case, even when the velocity is positive.

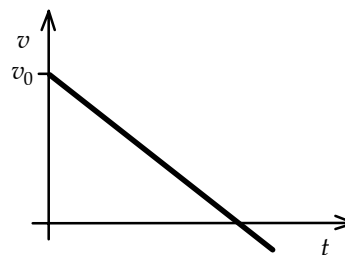


FIG. Q2.10

- Q2.11**
- | | | | | | |
|-----|-----------------------------------|-----|-------------------|-----|---------------|
| (a) | Accelerating East | (b) | Braking East | (c) | Cruising East |
| (d) | Braking West | (e) | Accelerating West | (f) | Cruising West |
| (g) | Stopped but starting to move East | | | | |
| (h) | Stopped but starting to move West | | | | |

Q2.12 No. Constant acceleration only. Yes. Zero is a constant.

Q2.13 The position does depend on the origin of the coordinate system. Assume that the cliff is 20 m tall, and that the stone reaches a maximum height of 10 m above the top of the cliff. If the origin is taken as the top of the cliff, then the maximum height reached by the stone would be 10 m. If the origin is taken as the bottom of the cliff, then the maximum height would be 30 m.

The velocity is independent of the origin. Since the *change* in position is used to calculate the instantaneous velocity in Equation 2.5, the choice of origin is arbitrary.

Q2.14 Once the objects leave the hand, both are in free fall, and both experience the same downward acceleration equal to the free-fall acceleration, $-g$.

Q2.15 They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity equal to v_i . This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

Q2.16 With $h = \frac{1}{2}gt^2$,

(a) $0.5h = \frac{1}{2}g(0.707t)^2$. The time is later than $0.5t$.

(b) The distance fallen is $0.25h = \frac{1}{2}g(0.5t)^2$. The elevation is $0.75h$, greater than $0.5h$.

- Q2.17** Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point.

SOLUTIONS TO PROBLEMS

Section 2.1 Position, Velocity, and Speed

P2.1 (a) $\bar{v} = \boxed{2.30 \text{ m/s}}$

(b) $v = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$

(c) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$

***P2.2** (a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{20 \text{ ft}}{1 \text{ yr}} \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) \left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}} \right) = \boxed{2 \times 10^{-7} \text{ m/s}}$ or in particularly windy times

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100 \text{ ft}}{1 \text{ yr}} \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) \left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}} \right) = \boxed{1 \times 10^{-6} \text{ m/s}}.$$

- (b) The time required must have been

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{3000 \text{ mi}}{10 \text{ mm/yr}} \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) \left(\frac{10^3 \text{ mm}}{1 \text{ m}} \right) = \boxed{5 \times 10^8 \text{ yr}}.$$

P2.3 (a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m}}{2 \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $\bar{v} = \frac{5 \text{ m}}{4 \text{ s}} = \boxed{1.2 \text{ m/s}}$

(c) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(d) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$

(e) $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0 \text{ m/s}}$

P2.4 $x = 10t^2$: For

$t(\text{s})$	=	2.0	2.1	3.0
$x(\text{m})$	=	40	44.1	90

(a) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{50 \text{ m}}{1.0 \text{ s}} = \boxed{50.0 \text{ m/s}}$

(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.1 \text{ m}}{0.1 \text{ s}} = \boxed{41.0 \text{ m/s}}$

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- P2.5** (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00 \text{ m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00 \text{ m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00 \text{ m/s})}$ and $t_2 = \frac{d}{(3.00 \text{ m/s})}$. The average speed is:

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d + d}{\frac{d}{(5.00 \text{ m/s})} + \frac{d}{(3.00 \text{ m/s})}} = \frac{2d}{\frac{(8.00 \text{ m/s})d}{(15.0 \text{ m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0 \text{ m}^2/\text{s}^2)}{8.00 \text{ m/s}} = \boxed{3.75 \text{ m/s}}$$

- (b) She starts and finishes at the same point A. With total displacement = 0, average velocity = $\boxed{0}$.

Section 2.2 Instantaneous Velocity and Speed

- P2.6** (a) At any time, t , the position is given by $x = (3.00 \text{ m/s}^2)t^2$.
Thus, at $t_i = 3.00 \text{ s}$: $x_i = (3.00 \text{ m/s}^2)(3.00 \text{ s})^2 = \boxed{27.0 \text{ m}}$.

- (b) At $t_f = 3.00 \text{ s} + \Delta t$: $x_f = (3.00 \text{ m/s}^2)(3.00 \text{ s} + \Delta t)^2$, or

$$x_f = \boxed{27.0 \text{ m} + (18.0 \text{ m/s})\Delta t + (3.00 \text{ m/s}^2)(\Delta t)^2}.$$

- (c) The instantaneous velocity at $t = 3.00 \text{ s}$ is:

$$v = \lim_{\Delta t \rightarrow 0} \left(\frac{x_f - x_i}{\Delta t} \right) = \lim_{\Delta t \rightarrow 0} (18.0 \text{ m/s} + (3.00 \text{ m/s}^2)\Delta t) = \boxed{18.0 \text{ m/s}}.$$

- P2.7** (a) at $t_i = 1.5 \text{ s}$, $x_i = 8.0 \text{ m}$ (Point A)
at $t_f = 4.0 \text{ s}$, $x_f = 2.0 \text{ m}$ (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0) \text{ m}}{(4 - 1.5) \text{ s}} = -\frac{6.0 \text{ m}}{2.5 \text{ s}} = \boxed{-2.4 \text{ m/s}}$$

- (b) The slope of the tangent line is found from points C and D. ($t_C = 1.0 \text{ s}$, $x_C = 9.5 \text{ m}$) and ($t_D = 3.5 \text{ s}$, $x_D = 0$),

$$v \cong \boxed{-3.8 \text{ m/s}}.$$

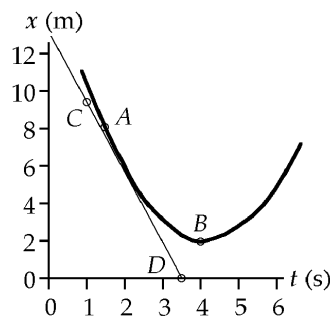
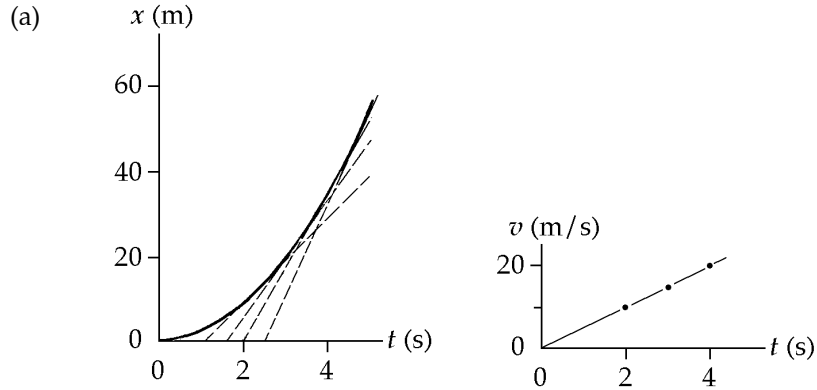


FIG. P2.7

- (c) The velocity is zero when x is a minimum. This is at $t \cong \boxed{4 \text{ s}}$.

P2.8



- (b) At $t = 5.0$ s, the slope is $v \cong \frac{58 \text{ m}}{2.5 \text{ s}} \cong \boxed{23 \text{ m/s}}$.
 At $t = 4.0$ s, the slope is $v \cong \frac{54 \text{ m}}{3 \text{ s}} \cong \boxed{18 \text{ m/s}}$.
 At $t = 3.0$ s, the slope is $v \cong \frac{49 \text{ m}}{3.4 \text{ s}} \cong \boxed{14 \text{ m/s}}$.
 At $t = 2.0$ s, the slope is $v \cong \frac{36 \text{ m}}{4.0 \text{ s}} \cong \boxed{9.0 \text{ m/s}}$.

(c) $\bar{a} = \frac{\Delta v}{\Delta t} \cong \frac{23 \text{ m/s}}{5.0 \text{ s}} \cong \boxed{4.6 \text{ m/s}^2}$

- (d) Initial velocity of the car was $\boxed{\text{zero}}$.

P2.9

(a) $v = \frac{(5-0) \text{ m}}{(1-0) \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $v = \frac{(5-10) \text{ m}}{(4-2) \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(c) $v = \frac{(5 \text{ m} - 5 \text{ m})}{(5 \text{ s} - 4 \text{ s})} = \boxed{0}$

(d) $v = \frac{0 - (-5 \text{ m})}{(8 \text{ s} - 7 \text{ s})} = \boxed{+5 \text{ m/s}}$

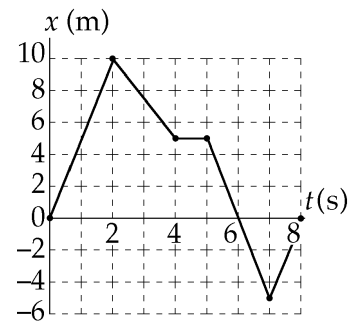


FIG. P2.9

*P2.10 Once it resumes the race, the hare will run for a time of

$$t = \frac{x_f - x_i}{v_x} = \frac{1000 \text{ m} - 800 \text{ m}}{8 \text{ m/s}} = 25 \text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f - x_i = (0.2 \text{ m/s})(25 \text{ s}) = \boxed{5.00 \text{ m}}.$$

Section 2.3 Acceleration

P2.11 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at: a = \frac{\Delta v}{\Delta t} = \frac{22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}.$$

P2.12 (a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + (2.00 \text{ m/s}^2)(10.0 \text{ s}) = \boxed{20.0 \text{ m/s}}.$$

Then $a = 0$ so v is constant from $t = 10.0 \text{ s}$ to $t = 15.0 \text{ s}$. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0 \text{ m/s} + (3.00 \text{ m/s}^2)(5.00 \text{ s}) = \boxed{5.00 \text{ m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} (2.00 \text{ m/s}^2)(10.0 \text{ s})^2 = 100 \text{ m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 100 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + 0 = 200 \text{ m}.$$

And at $t = 20.0 \text{ s}$,

$$x_f = x_i + v_i t + \frac{1}{2} a t^2 = 200 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2} (-3.00 \text{ m/s}^2)(5.00 \text{ s})^2 = \boxed{262 \text{ m}}.$$

***P2.13** (a) The average speed during a time interval Δt is $\bar{v} = \frac{\text{distance traveled}}{\Delta t}$. During the first quarter mile segment, Secretariat's average speed was

$$\bar{v}_1 = \frac{0.250 \text{ mi}}{25.2 \text{ s}} = \frac{1320 \text{ ft}}{25.2 \text{ s}} = \boxed{52.4 \text{ ft/s}} \quad (35.6 \text{ mi/h}).$$

During the second quarter mile segment,

$$\bar{v}_2 = \frac{1320 \text{ ft}}{24.0 \text{ s}} = \boxed{55.0 \text{ ft/s}} \quad (37.4 \text{ mi/h}).$$

For the third quarter mile of the race,

$$\bar{v}_3 = \frac{1320 \text{ ft}}{23.8 \text{ s}} = \boxed{55.5 \text{ ft/s}} \quad (37.7 \text{ mi/h}),$$

and during the final quarter mile,

$$\bar{v}_4 = \frac{1320 \text{ ft}}{23.0 \text{ s}} = \boxed{57.4 \text{ ft/s}} \quad (39.0 \text{ mi/h}).$$

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- (b) Assuming that $v_f = \bar{v}_4$ and recognizing that $v_i = 0$, the average acceleration during the race was

$$\bar{a} = \frac{v_f - v_i}{\text{total elapsed time}} = \frac{57.4 \text{ ft/s} - 0}{(25.2 + 24.0 + 23.8 + 23.0) \text{ s}} = \boxed{0.598 \text{ ft/s}^2}.$$

- P2.14** (a) Acceleration is the slope of the graph of v vs t .

For $0 < t < 5.00 \text{ s}$, $a = 0$.

For $15.0 \text{ s} < t < 20.0 \text{ s}$, $a = 0$.

For $5.0 \text{ s} < t < 15.0 \text{ s}$, $a = \frac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60 \text{ m/s}^2$$

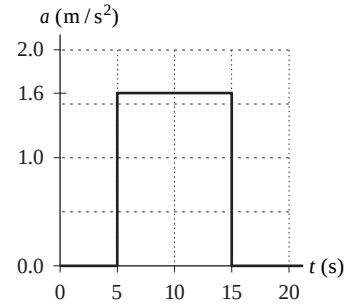


FIG. P2.14

We can plot $a(t)$ as shown.

(b)
$$a = \frac{v_f - v_i}{t_f - t_i}$$

- (i) For $5.00 \text{ s} < t < 15.0 \text{ s}$, $t_i = 5.00 \text{ s}$, $v_i = -8.00 \text{ m/s}$,

$$t_f = 15.0 \text{ s}$$

$$v_f = 8.00 \text{ m/s}$$

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60 \text{ m/s}^2}.$$

- (ii) $t_i = 0$, $v_i = -8.00 \text{ m/s}$, $t_f = 20.0 \text{ s}$, $v_f = 8.00 \text{ m/s}$

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800 \text{ m/s}^2}$$

P2.15 $x = 2.00 + 3.00t - t^2$, $v = \frac{dx}{dt} = 3.00 - 2.00t$, $a = \frac{dv}{dt} = -2.00$

At $t = 3.00 \text{ s}$:

(a) $x = (2.00 + 9.00 - 9.00) \text{ m} = \boxed{2.00 \text{ m}}$

(b) $v = (3.00 - 6.00) \text{ m/s} = \boxed{-3.00 \text{ m/s}}$

(c) $a = \boxed{-2.00 \text{ m/s}^2}$

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P2.16 (a) At $t = 2.00 \text{ s}$, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00] \text{ m} = 11.0 \text{ m}$.

At $t = 3.00 \text{ s}$, $x = [3.00(3.00)^2 - 2.00(3.00) + 3.00] \text{ m} = 24.0 \text{ m}$

so

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}(3.00t^2 - 2.00t + 3.00) = (6.00t - 2.00) \text{ m/s}$$

At $t = 2.00 \text{ s}$, $v = [6.00(2.00) - 2.00] \text{ m/s} = \boxed{10.0 \text{ m/s}}$.

At $t = 3.00 \text{ s}$, $v = [6.00(3.00) - 2.00] \text{ m/s} = \boxed{16.0 \text{ m/s}}$.

(c) $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$

(d) At all times $a = \frac{d}{dt}(6.00 - 2.00) = \boxed{6.00 \text{ m/s}^2}$. (This includes both $t = 2.00 \text{ s}$ and $t = 3.00 \text{ s}$).

P2.17 (a) $a = \frac{\Delta v}{\Delta t} = \frac{8.00 \text{ m/s}}{6.00 \text{ s}} = \boxed{1.3 \text{ m/s}^2}$

(b) Maximum positive acceleration is at $t = 3 \text{ s}$, and is approximately $\boxed{2 \text{ m/s}^2}$.

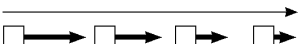
(c) $a = 0$, at $\boxed{t = 6 \text{ s}}$, and also for $\boxed{t > 10 \text{ s}}$.

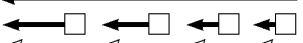
(d) Maximum negative acceleration is at $t = 8 \text{ s}$, and is approximately $\boxed{-1.5 \text{ m/s}^2}$.

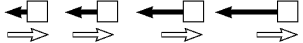
Section 2.4 Motion Diagrams

P2.18 (a) 
 $\rightarrow$ = reading order
 $\Rightarrow$ = velocity
 $\Rightarrow\Rightarrow$ = acceleration

(b) 

(c) 

(d) 

(e) 

continued on next page

- (f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.
 Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

Section 2.5 One-Dimensional Motion with Constant Acceleration

P2.19 From $v_f^2 = v_i^2 + 2ax$, we have $(10.97 \times 10^3 \text{ m/s})^2 = 0 + 2a(220 \text{ m})$, so that $a = 2.74 \times 10^5 \text{ m/s}^2$ which is $a = 2.79 \times 10^4 \text{ times } g$.

P2.20 (a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$ becomes $40 \text{ m} = \frac{1}{2}(v_i + 2.80 \text{ m/s})(8.50 \text{ s})$ which yields $v_i = 6.61 \text{ m/s}$.

(b) $a = \frac{v_f - v_i}{t} = \frac{2.80 \text{ m/s} - 6.61 \text{ m/s}}{8.50 \text{ s}} = -0.448 \text{ m/s}^2$

P2.21 Given $v_i = 12.0 \text{ cm/s}$ when $x_i = 3.00 \text{ cm}(t = 0)$, and at $t = 2.00 \text{ s}$, $x_f = -5.00 \text{ cm}$,

$$x_f - x_i = v_i t + \frac{1}{2} a t^2: -5.00 - 3.00 = 12.0(2.00) + \frac{1}{2} a(2.00)^2$$

$$-8.00 = 24.0 + 2a \quad a = -\frac{32.0}{2} = -16.0 \text{ cm/s}^2.$$

***P2.22** (a) Let i be the state of moving at 60 mi/h and f be at rest

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (60 \text{ mi/h})^2 + 2a_x(121 \text{ ft} - 0) \left(\frac{1 \text{ mi}}{5280 \text{ ft}} \right)$$

$$a_x = \frac{-3600 \text{ mi}}{242 \text{ h}^2} \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = -21.8 \text{ mi/h} \cdot \text{s}$$

$$= -21.8 \text{ mi/h} \cdot \text{s} \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = -9.75 \text{ m/s}^2.$$

(b) Similarly,

$$0 = (80 \text{ mi/h})^2 + 2a_x(211 \text{ ft} - 0)$$

$$a_x = -\frac{6400(5280)}{422(3600)} \text{ mi/h} \cdot \text{s} = -22.2 \text{ mi/h} \cdot \text{s} = -9.94 \text{ m/s}^2.$$

(c) Let i be moving at 80 mi/h and f be moving at 60 mi/h .

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$(60 \text{ mi/h})^2 = (80 \text{ mi/h})^2 + 2a_x(211 \text{ ft} - 121 \text{ ft})$$

$$a_x = -\frac{2800(5280)}{2(90)(3600)} \text{ mi/h} \cdot \text{s} = -22.8 \text{ mi/h} \cdot \text{s} = -10.2 \text{ m/s}^2.$$

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- *P2.23** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy:

$$x_i = 0, x_f = 100 \text{ m}, v_{xi} = 30 \text{ m/s}, v_{xf} = ?, a_x = -3.5 \text{ m/s}^2, t = ?$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2:$$

$$100 \text{ m} = 0 + (30 \text{ m/s})t + \frac{1}{2}(-3.5 \text{ m/s}^2)t^2$$

$$(1.75 \text{ m/s}^2)t^2 - (30 \text{ m/s})t + 100 \text{ m} = 0.$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

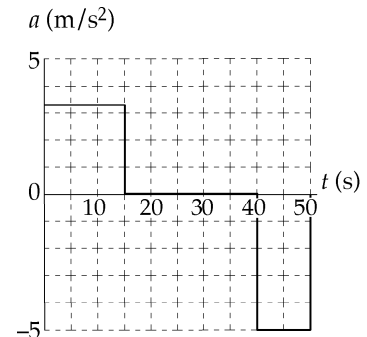
$$t = \frac{30 \text{ m/s} \pm \sqrt{900 \text{ m}^2/\text{s}^2 - 4(1.75 \text{ m/s}^2)(100 \text{ m})}}{2(1.75 \text{ m/s}^2)} = \frac{30 \text{ m/s} \pm 14.1 \text{ m/s}}{3.5 \text{ m/s}^2} = 12.6 \text{ s or } \boxed{4.53 \text{ s}}.$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30 \text{ m/s} - (3.5 \text{ m/s}^2)4.53 \text{ s} = \boxed{14.1 \text{ m/s}}$

- P2.24** (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s.

$$\begin{aligned} \Delta x &= \frac{1}{2}(50 \text{ m/s})(15 \text{ s}) + (50 \text{ m/s})(40 - 15) \text{ s} \\ &\quad + \frac{1}{2}(50 \text{ m/s})(10 \text{ s}) \\ \Delta x &= \boxed{1875 \text{ m}} \end{aligned}$$



- (b) From $t = 10 \text{ s}$ to $t = 40 \text{ s}$, displacement is

$$\Delta x = \frac{1}{2}(50 \text{ m/s} + 33 \text{ m/s})(5 \text{ s}) + (50 \text{ m/s})(25 \text{ s}) = \boxed{1457 \text{ m}}.$$

(c) $0 \leq t \leq 15 \text{ s}: a_1 = \frac{\Delta v}{\Delta t} = \frac{(50 - 0) \text{ m/s}}{15 \text{ s} - 0} = \boxed{3.3 \text{ m/s}^2}$

$15 \text{ s} < t < 40 \text{ s}: \boxed{a_2 = 0}$

$40 \text{ s} \leq t \leq 50 \text{ s}: a_3 = \frac{\Delta v}{\Delta t} = \frac{(0 - 50) \text{ m/s}}{50 \text{ s} - 40 \text{ s}} = \boxed{-5.0 \text{ m/s}^2}$

FIG. P2.24

continued on next page

- (d) (i) $x_1 = 0 + \frac{1}{2}a_1t^2 = \frac{1}{2}(3.3 \text{ m/s}^2)t^2$ or $x_1 = (1.67 \text{ m/s}^2)t^2$
- (ii) $x_2 = \frac{1}{2}(15 \text{ s})[50 \text{ m/s} - 0] + (50 \text{ m/s})(t - 15 \text{ s})$ or $x_2 = (50 \text{ m/s})t - 375 \text{ m}$
- (iii) For $40 \text{ s} \leq t \leq 50 \text{ s}$,

$$x_3 = \left(\begin{array}{l} \text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40 \text{ s} \end{array} \right) + \frac{1}{2}a_3(t - 40 \text{ s})^2 + (50 \text{ m/s})(t - 40 \text{ s})$$

or

$$x_3 = 375 \text{ m} + 1\,250 \text{ m} + \frac{1}{2}(-5.0 \text{ m/s}^2)(t - 40 \text{ s})^2 + (50 \text{ m/s})(t - 40 \text{ s})$$

which reduces to

$$x_3 = (250 \text{ m/s})t - (2.5 \text{ m/s}^2)t^2 - 4\,375 \text{ m}.$$

(e) $\bar{v} = \frac{\text{total displacement}}{\text{total elapsed time}} = \frac{1\,875 \text{ m}}{50 \text{ s}} = 37.5 \text{ m/s}$

- P2.25** (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00 \text{ m}$, $v_i = 3.00 \text{ m/s}$, and $a = -8.00 \text{ m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8} \text{ s}$. The position at this time is:

$$x = 2.00 \text{ m} + (3.00 \text{ m/s})\left(\frac{3}{8} \text{ s}\right) - (4.00 \text{ m/s}^2)\left(\frac{3}{8} \text{ s}\right)^2 = 2.56 \text{ m}.$$

- (b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s}$$

and the velocity is $v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)\left(\frac{3}{4} \text{ s}\right) = -3.00 \text{ m/s}$.

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***P2.26** The time for the Ford to slow down we find from

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$

$$t = \frac{2\Delta x}{v_{xi} + v_{xf}} = \frac{2(250 \text{ m})}{71.5 \text{ m/s} + 0} = 6.99 \text{ s}.$$

Its time to speed up is similarly

$$t = \frac{2(350 \text{ m})}{0 + 71.5 \text{ m/s}} = 9.79 \text{ s}.$$

The whole time it is moving at less than maximum speed is $6.99 \text{ s} + 5.00 \text{ s} + 9.79 \text{ s} = 21.8 \text{ s}$. The Mercedes travels

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(71.5 + 71.5)(\text{m/s})(21.8 \text{ s})$$

$$= 1558 \text{ m}$$

while the Ford travels $250 + 350 \text{ m} = 600 \text{ m}$, to fall behind by $1558 \text{ m} - 600 \text{ m} = \boxed{958 \text{ m}}$.

P2.27 (a) $v_i = 100 \text{ m/s}$, $a = -5.00 \text{ m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a(x_f - x_i)$ so $0 = (100)^2 - 2(5.00)(x_f - 0)$. Thus $x_f = 1000 \text{ m}$ and $t = \boxed{20.0 \text{ s}}$.

(b) At this acceleration the plane would overshoot the runway: No.

P2.28 (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0 \text{ m/s}$, $a = -2.00 \text{ m/s}^2$. Use these values in the general equation

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to find

$$x_f = 0 + (30.0t \text{ m/s}) + \frac{1}{2}(-2.00 \text{ m/s}^2)t^2$$

when t is in seconds

$$\boxed{x_f = (30.0t - t^2) \text{ m}}.$$

To find an equation for the velocity, use $v_f = v_i + at = 30.0 \text{ m/s} + (-2.00 \text{ m/s}^2)t$,

$$\boxed{v_f = (30.0 - 2.00t) \text{ m/s}}.$$

(b) The distance of travel x_f becomes a maximum, $x_{\max}$, when $v_f = 0$ (turning point in the motion). Use the expressions found in part (a) for v_f to find the value of t when x_f has its maximum value:

From $v_f = (30.0 - 2.00t) \text{ m/s}$, $v_f = 0$ when $t = 15.0 \text{ s}$. Then

$$x_{\max} = (30.0t - t^2) \text{ m} = (30.0)(15.0) - (15.0)^2 = \boxed{225 \text{ m}}.$$

P2.29 In the simultaneous equations:

$$\left\{ \begin{array}{l} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{array} \right\} \text{ we have } \left\{ \begin{array}{l} v_{xf} = v_{xi} - (5.60 \text{ m/s}^2)(4.20 \text{ s}) \\ 62.4 \text{ m} = \frac{1}{2}(v_{xi} + v_{xf})(4.20 \text{ s}) \end{array} \right\}.$$

So substituting for v_{xi} gives $62.4 \text{ m} = \frac{1}{2}[v_{xf} + (56.0 \text{ m/s}^2)(4.20 \text{ s}) + v_{xf}](4.20 \text{ s})$

$$14.9 \text{ m/s} = v_{xf} + \frac{1}{2}(5.60 \text{ m/s}^2)(4.20 \text{ s}).$$

Thus

$$v_{xf} = \boxed{3.10 \text{ m/s}}.$$

P2.30 Take any two of the standard four equations, such as $\left\{ \begin{array}{l} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{array} \right\}$. Solve one for v_{xi} , and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}(v_{xf} - a_x t + v_{xf})t.$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}.$$

Back in problem 29, $62.4 \text{ m} = v_{xf}(4.20 \text{ s}) - \frac{1}{2}(-5.60 \text{ m/s}^2)(4.20 \text{ s})^2$

$$v_{xf} = \frac{62.4 \text{ m} - 49.4 \text{ m}}{4.20 \text{ s}} = \boxed{3.10 \text{ m/s}}.$$

P2.31 (a) $a = \frac{v_f - v_i}{t} = \frac{632 \left(\frac{5280}{3600} \right)}{1.40} = \boxed{-662 \text{ ft/s}^2} = -202 \text{ m/s}^2$

(b) $x_f = v_i t + \frac{1}{2} a t^2 = (632) \left(\frac{5280}{3600} \right) (1.40) - \frac{1}{2} (662) (1.40)^2 = \boxed{649 \text{ ft}} = \boxed{198 \text{ m}}$

34 Motion in One Dimension

P2.32 (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$. Solving for t yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ m/s}^2} = 10.0 \text{ s}.$$

The total time is thus

$$10.0 \text{ s} + 20.0 \text{ s} + 5.00 \text{ s} = \boxed{35.0 \text{ s}}.$$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2} \right)(10.0) = 100 \text{ m}.$$

With a being 0 for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2} a t^2 = (20.0)(20.0) + 0 = 400 \text{ m}.$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2} \right)(5.00) = 50.0 \text{ m}.$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550 \text{ m}$, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$.

P2.33 We have $v_i = 2.00 \times 10^4 \text{ m/s}$, $v_f = 6.00 \times 10^6 \text{ m/s}$, $x_f - x_i = 1.50 \times 10^{-2} \text{ m}$.

$$(a) \quad x_f - x_i = \frac{1}{2}(v_i + v_f)t: t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2} \text{ m})}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}} = \boxed{4.98 \times 10^{-9} \text{ s}}$$

$$(b) \quad v_f^2 = v_i^2 + 2a_x(x_f - x_i):$$

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6 \text{ m/s})^2 - (2.00 \times 10^4 \text{ m/s})^2}{2(1.50 \times 10^{-2} \text{ m})} = \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

***P2.34** (a) $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) : \left[0.01(3 \times 10^8 \text{ m/s})\right]^2 = 0 + 2a_x(40 \text{ m})$

$$a_x = \frac{(3 \times 10^6 \text{ m/s})^2}{80 \text{ m}} = \boxed{1.12 \times 10^{11} \text{ m/s}^2}$$

(b) We must find separately the time t_1 for speeding up and the time t_2 for coasting:

$$x_f - x_i = \frac{1}{2}(v_{xf} + v_{xi})t_1 : 40 \text{ m} = \frac{1}{2}(3 \times 10^6 \text{ m/s} + 0)t_1$$

$$t_1 = 2.67 \times 10^{-5} \text{ s}$$

$$x_f - x_i = \frac{1}{2}(v_{xf} + v_{xi})t_2 : 60 \text{ m} = \frac{1}{2}(3 \times 10^6 \text{ m/s} + 3 \times 10^6 \text{ m/s})t_2$$

$$t_2 = 2.00 \times 10^{-5} \text{ s}$$

$$\text{total time} = \boxed{4.67 \times 10^{-5} \text{ s}}.$$

***P2.35** (a) Along the time axis of the graph shown, let $i = 0$ and $f = t_m$. Then $v_{xf} = v_{xi} + a_x t$ gives $v_c = 0 + a_m t_m$

$$\boxed{a_m = \frac{v_c}{t_m}}.$$

(b) The displacement between 0 and t_m is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\frac{v_c}{t_m}t_m^2 = \frac{1}{2}v_c t_m.$$

The displacement between t_m and t_0 is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = v_c(t_0 - t_m) + 0.$$

The total displacement is

$$\Delta x = \frac{1}{2}v_c t_m + v_c t_0 - v_c t_m = \boxed{v_c \left(t_0 - \frac{1}{2}t_m\right)}.$$

(c) For constant v_c and t_0 , Δx is minimized by maximizing t_m to $t_m = t_0$. Then

$$\Delta x_{\min} = v_c \left(t_0 - \frac{1}{2}t_0\right) = \boxed{\frac{v_c t_0}{2}}.$$

(e) This is realized by having the servo motor on all the time.

(d) We maximize Δx by letting t_m approach zero. In the limit $\Delta x = v_c(t_0 - 0) = \boxed{v_c t_0}$.

(e) This cannot be attained because the acceleration must be finite.

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- *P2.36** Let the glider enter the photogate with velocity v_i and move with constant acceleration a . For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}at^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

- (a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d.$$

$$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d} \text{ and this is not equal to } v_d \text{ unless } a = 0.$$

- (b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is equal to v_d as determined above.

- P2.37** (a) Take initial and final points at top and bottom of the incline. If the ball starts from rest,

$$v_i = 0, a = 0.500 \text{ m/s}^2, x_f - x_i = 9.00 \text{ m}.$$

Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500 \text{ m/s}^2)(9.00 \text{ m})$$

$$v_f = \boxed{3.00 \text{ m/s}}.$$

- (b) $x_f - x_i = v_i t + \frac{1}{2}at^2$

$$9.00 = 0 + \frac{1}{2}(0.500 \text{ m/s}^2)t^2$$

$$t = \boxed{6.00 \text{ s}}$$

- (c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00 \text{ m/s}, v_f = 0, x_f - x_i = 15.00 \text{ m}.$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \text{ gives}$$

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{0 - (3.00 \text{ m/s})^2}{2(15.0 \text{ m})} = \boxed{-0.300 \text{ m/s}^2}.$$

- (d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second:
 $v_i = 3.00 \text{ m/s}, x_f - x_i = 8.00 \text{ m}, a = -0.300 \text{ m/s}^2$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00 \text{ m/s})^2 + 2(-0.300 \text{ m/s}^2)(8.00 \text{ m}) = 4.20 \text{ m}^2/\text{s}^2$$

$$v_f = \boxed{2.05 \text{ m/s}}.$$

- P2.38** Take the original point to be when Sue notices the van. Choose the origin of the x -axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0 \text{ m/s}$, $a_s = -2.00 \text{ m/s}^2$ so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)t^2.$$

For the van, $x_{iv} = 155 \text{ m}$, $v_{iv} = 5.00 \text{ m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00 \text{ m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$\begin{aligned} 30.0t_c - t_c^2 &= 155 + 5.00t_c \\ 0 &= t_c^2 - 25.0t_c + 155. \end{aligned}$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6 \text{ s or } \boxed{11.4 \text{ s}}.$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155 \text{ m} + (5.00 \text{ m/s})(11.4 \text{ s}) = \boxed{212 \text{ m}}.$$

- *P2.39** As in the algebraic solution to Example 2.8, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and

$$x_{\text{trooper}} = 1.5t^2.$$

They intersect at

$$t = \boxed{31 \text{ s}}.$$

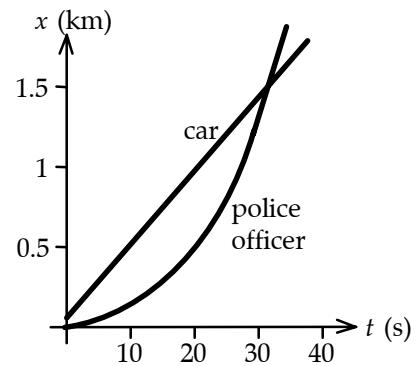


FIG. P2.39

Section 2.6 Freely Falling Objects

P2.40 Choose the origin ($y = 0$, $t = 0$) at the starting point of the ball and take upward as positive. Then $y_i = 0$, $v_i = 0$, and $a = -g = -9.80 \text{ m/s}^2$. The position and the velocity at time t become:

$$y_f - y_i = v_i t + \frac{1}{2} a t^2: y_f = -\frac{1}{2} g t^2 = -\frac{1}{2} (9.80 \text{ m/s}^2) t^2$$

and

$$v_f = v_i + a t: v_f = -g t = -(9.80 \text{ m/s}^2) t.$$

(a) at $t = 1.00 \text{ s}$: $y_f = -\frac{1}{2} (9.80 \text{ m/s}^2) (1.00 \text{ s})^2 = \boxed{-4.90 \text{ m}}$

at $t = 2.00 \text{ s}$: $y_f = -\frac{1}{2} (9.80 \text{ m/s}^2) (2.00 \text{ s})^2 = \boxed{-19.6 \text{ m}}$

at $t = 3.00 \text{ s}$: $y_f = -\frac{1}{2} (9.80 \text{ m/s}^2) (3.00 \text{ s})^2 = \boxed{-44.1 \text{ m}}$

(b) at $t = 1.00 \text{ s}$: $v_f = -(9.80 \text{ m/s}^2) (1.00 \text{ s}) = \boxed{-9.80 \text{ m/s}}$

at $t = 2.00 \text{ s}$: $v_f = -(9.80 \text{ m/s}^2) (2.00 \text{ s}) = \boxed{-19.6 \text{ m/s}}$

at $t = 3.00 \text{ s}$: $v_f = -(9.80 \text{ m/s}^2) (3.00 \text{ s}) = \boxed{-29.4 \text{ m/s}}$

P2.41 Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80 \text{ m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2 = v_i^2 + 2a(y_f - y_i): \quad 0 = v_i^2 - 2(9.80 \text{ m/s}^2)(1\,609 \text{ m})$$

$$v_i = 178 \text{ m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f - y_i = v_i t + \frac{1}{2} a t^2: 0 = (178 \text{ m/s}) t - \frac{1}{2} (-9.80 \text{ m/s}^2) t^2.$$

The root $t = 0$ describes launch; the other root, $t = 36.2 \text{ s}$, describes his flight time. His rate of pay may then be found from

$$\text{pay rate} = \frac{\$1.00}{36.2 \text{ s}} = (0.0276 \text{ \$/s})(3\,600 \text{ s/h}) = \boxed{\$99.3/\text{h}}.$$

We have assumed that the workman's flight time, "a mile", and "a dollar", were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times. Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

P2.42 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -(4.90 \text{ m/s}^2)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}.$$

Solving for t ,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.$$

Using only the positive value for t , we find that $t = \boxed{1.79 \text{ s}}$.

P2.43 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0 \text{ m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$$v_f = \boxed{4.68 \text{ m/s downward}}$$

P2.44 The bill starts from rest $v_i = 0$ and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2}gt^2 = 0 - (4.90 \text{ m/s}^2)(0.20 \text{ s})^2 = -0.20 \text{ m}.$$

This distance is about twice the distance between the center of the bill and its top edge ($\cong 8 \text{ cm}$).

Thus, David will be unsuccessful.

***P2.45** (a) From $\Delta y = v_i t + \frac{1}{2}at^2$ with $v_i = 0$, we have

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-23 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{2.17 \text{ s}}.$$

(b) The final velocity is $v_f = 0 + (-9.80 \text{ m/s}^2)(2.17 \text{ s}) = \boxed{-21.2 \text{ m/s}}$.

(c) The time take for the sound of the impact to reach the spectator is

$$t_{\text{sound}} = \frac{\Delta y}{v_{\text{sound}}} = \frac{23 \text{ m}}{340 \text{ m/s}} = 6.76 \times 10^{-2} \text{ s},$$

so the total elapsed time is $t_{\text{total}} = 2.17 \text{ s} + 6.76 \times 10^{-2} \text{ s} \approx \boxed{2.23 \text{ s}}$.

P2.46 At any time t , the position of the ball released from rest is given by $y_1 = h - \frac{1}{2}gt^2$. At time t , the position of the ball thrown vertically upward is described by $y_2 = v_i t - \frac{1}{2}gt^2$. The time at which the first ball has a position of $y_1 = \frac{h}{2}$ is found from the first equation as $\frac{h}{2} = h - \frac{1}{2}gt^2$, which yields $t = \sqrt{\frac{h}{g}}$. To require that the second ball have a position of $y_2 = \frac{h}{2}$ at this time, use the second equation to obtain $\frac{h}{2} = v_i \sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\sqrt{\frac{h}{g}}\right)^2$. This gives the required initial upward velocity of the second ball as $\boxed{v_i = \sqrt{gh}}$.

P2.47 (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00$ s, $g = 9.80$ m/s². Therefore,

$$v_i = gt = (9.80 \text{ m/s}^2)(3.00 \text{ s}) = \boxed{29.4 \text{ m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4 \text{ m/s})(3.00 \text{ s}) = \boxed{44.1 \text{ m}}$$

***P2.48** (a) Consider the upward flight of the arrow.

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y(y_f - y_i) \\ 0 &= (100 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)\Delta y \\ \Delta y &= \frac{10\,000 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = \boxed{510 \text{ m}} \end{aligned}$$

(b) Consider the whole flight of the arrow.

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ 0 &= 0 + (100 \text{ m/s})t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2 \end{aligned}$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100 \text{ m/s}}{4.9 \text{ m/s}^2} = \boxed{20.4 \text{ s}}.$$

P2.49 Time to fall 3.00 m is found from Eq. 2.12 with $v_i = 0$, $3.00 \text{ m} = \frac{1}{2}(9.80 \text{ m/s}^2)t^2$, $t = 0.782$ s.

(a) With the horse galloping at 10.0 m/s, the horizontal distance is $vt = \boxed{7.82 \text{ m}}$.

(b) $t = \boxed{0.782 \text{ s}}$

P2.50 Take downward as the positive y direction.

(a) While the woman was in free fall,

$$\Delta y = 144 \text{ ft}, v_i = 0, \text{ and } a = g = 32.0 \text{ ft/s}^2.$$

Thus, $\Delta y = v_i t + \frac{1}{2} a t^2 \rightarrow 144 \text{ ft} = 0 + (16.0 \text{ ft/s}^2) t^2$ giving $t_{\text{fall}} = 3.00 \text{ s}$. Her velocity just before impact is:

$$v_f = v_i + g t = 0 + (32.0 \text{ ft/s}^2)(3.00 \text{ s}) = \boxed{96.0 \text{ ft/s}}.$$

(b) While crushing the box, $v_i = 96.0 \text{ ft/s}$, $v_f = 0$, and $\Delta y = 18.0 \text{ in.} = 1.50 \text{ ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0 \text{ ft/s})^2}{2(1.50 \text{ ft})} = -3.07 \times 10^3 \text{ ft/s}^2, \text{ or } \boxed{a = 3.07 \times 10^3 \text{ ft/s}^2 \text{ upward}}.$$

(c) Time to crush box: $\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50 \text{ ft})}{0 + 96.0 \text{ ft/s}} \text{ or } \boxed{\Delta t = 3.13 \times 10^{-2} \text{ s}}.$

P2.51 $y = 3.00t^3$: At $t = 2.00 \text{ s}$, $y = 3.00(2.00)^3 = 24.0 \text{ m}$ and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2} g t^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for t , (only positive values of t count), $\boxed{t = 7.96 \text{ s}}.$

***P2.52** Consider the last 30 m of fall. We find its speed 30 m above the ground:

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ 0 &= 30 \text{ m} + v_{yi}(1.5 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(1.5 \text{ s})^2 \\ v_{yi} &= \frac{-30 \text{ m} + 11.0 \text{ m}}{1.5 \text{ s}} = -12.6 \text{ m/s}. \end{aligned}$$

Now consider the portion of its fall above the 30 m point. We assume it starts from rest

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y(y_f - y_i) \\ (-12.6 \text{ m/s})^2 &= 0 + 2(-9.8 \text{ m/s}^2)\Delta y \\ \Delta y &= \frac{160 \text{ m}^2/\text{s}^2}{-19.6 \text{ m/s}^2} = -8.16 \text{ m}. \end{aligned}$$

Its original height was then $30 \text{ m} + |-8.16 \text{ m}| = \boxed{38.2 \text{ m}}.$

Section 2.7 Kinematic Equations Derived from Calculus

P2.53 (a) $J = \frac{da}{dt} = \text{constant}$

$$da = Jdt$$

$$a = J \int dt = Jt + c_1$$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$. Therefore, $\boxed{a = Jt + a_i}$

$$a = \frac{dv}{dt}$$

$$dv = a dt$$

$$v = \int a dt = \int (Jt + a_i) dt = \frac{1}{2} Jt^2 + a_i t + c_2$$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2} Jt^2 + a_i t + v_i}$

$$v = \frac{dx}{dt}$$

$$dx = v dt$$

$$x = \int v dt = \int \left(\frac{1}{2} Jt^2 + a_i t + v_i \right) dt$$

$$x = \frac{1}{6} Jt^3 + \frac{1}{2} a_i t^2 + v_i t + c_3$$

$$x = x_i$$

when $t = 0$, so $c_3 = x_i$. Therefore, $\boxed{x = \frac{1}{6} Jt^3 + \frac{1}{2} a_i t^2 + v_i t + x_i}$.

(b) $a^2 = (Jt + a_i)^2 = J^2 t^2 + a_i^2 + 2Ja_i t$

$$a^2 = a_i^2 + (J^2 t^2 + 2Ja_i t)$$

$$a^2 = a_i^2 + 2J \left(\frac{1}{2} Jt^2 + a_i t \right)$$

Recall the expression for v : $v = \frac{1}{2} Jt^2 + a_i t + v_i$. So $(v - v_i) = \frac{1}{2} Jt^2 + a_i t$. Therefore,

$$\boxed{a^2 = a_i^2 + 2J(v - v_i)}$$

P2.54 (a) See the graphs at the right.

Choose $x = 0$ at $t = 0$.

$$\text{At } t = 3 \text{ s, } x = \frac{1}{2}(8 \text{ m/s})(3 \text{ s}) = 12 \text{ m.}$$

$$\text{At } t = 5 \text{ s, } x = 12 \text{ m} + (8 \text{ m/s})(2 \text{ s}) = 28 \text{ m.}$$

$$\text{At } t = 7 \text{ s, } x = 28 \text{ m} + \frac{1}{2}(8 \text{ m/s})(2 \text{ s}) = 36 \text{ m.}$$

(b) For $0 < t < 3 \text{ s}$, $a = \frac{8 \text{ m/s}}{3 \text{ s}} = 2.67 \text{ m/s}^2$.

For $3 < t < 5 \text{ s}$, $a = 0$.

(c) For $5 \text{ s} < t < 9 \text{ s}$, $a = -\frac{16 \text{ m/s}}{4 \text{ s}} = \boxed{-4 \text{ m/s}^2}$.

(d) At $t = 6 \text{ s}$, $x = 28 \text{ m} + (6 \text{ m/s})(1 \text{ s}) = \boxed{34 \text{ m}}$.

(e) At $t = 9 \text{ s}$, $x = 36 \text{ m} + \frac{1}{2}(-8 \text{ m/s})(2 \text{ s}) = \boxed{28 \text{ m}}$.

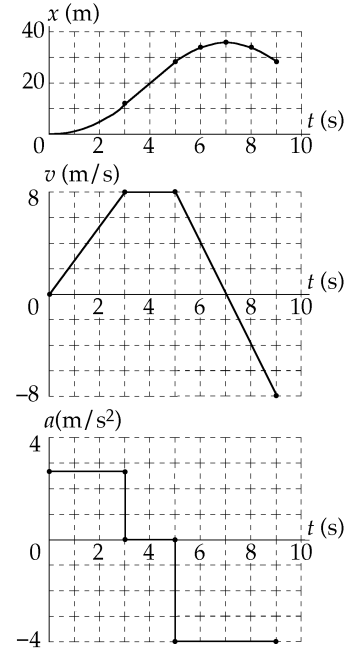


FIG. P2.54

P2.55 (a) $a = \frac{dv}{dt} = \frac{d}{dt}[-5.00 \times 10^7 t^2 + 3.00 \times 10^5 t]$

$$\boxed{a = -(10.0 \times 10^7 \text{ m/s}^3)t + 3.00 \times 10^5 \text{ m/s}^2}$$

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x - 0 = \int_0^t v dt = \int_0^t (-5.00 \times 10^7 t^2 + 3.00 \times 10^5 t) dt$$

$$x = -5.00 \times 10^7 \frac{t^3}{3} + 3.00 \times 10^5 \frac{t^2}{2}$$

$$\boxed{x = -(1.67 \times 10^7 \text{ m/s}^3)t^3 + (1.50 \times 10^5 \text{ m/s}^2)t^2}$$

(b) The bullet escapes when $a = 0$, at $-(10.0 \times 10^7 \text{ m/s}^3)t + 3.00 \times 10^5 \text{ m/s}^2 = 0$

$$t = \frac{3.00 \times 10^5 \text{ s}}{10.0 \times 10^7} = \boxed{3.00 \times 10^{-3} \text{ s}}$$

(c) New $v = (-5.00 \times 10^7)(3.00 \times 10^{-3})^2 + (3.00 \times 10^5)(3.00 \times 10^{-3})$

$$v = -450 \text{ m/s} + 900 \text{ m/s} = \boxed{450 \text{ m/s}}$$

(d) $x = -(1.67 \times 10^7)(3.00 \times 10^{-3})^3 + (1.50 \times 10^5)(3.00 \times 10^{-3})^2$

$$x = -0.450 \text{ m} + 1.35 \text{ m} = \boxed{0.900 \text{ m}}$$

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P2.56 $a = \frac{dv}{dt} = -3.00v^2$, $v_i = 1.50 \text{ m/s}$

Solving for v , $\frac{dv}{dt} = -3.00v^2$

$$\int_{v=v_i}^v v^{-2} dv = -3.00 \int_{t=0}^t dt$$

$$-\frac{1}{v} + \frac{1}{v_i} = -3.00t \text{ or } 3.00t = \frac{1}{v} - \frac{1}{v_i}.$$

When $v = \frac{v_i}{2}$, $t = \frac{1}{3.00v_i} = \boxed{0.222 \text{ s}}$.

Additional Problems

- *P2.57** The distance the car travels at constant velocity, v_0 , during the reaction time is $(\Delta x)_1 = v_0 \Delta t_r$. The time for the car to come to rest, from initial velocity v_0 , after the brakes are applied is

$$t_2 = \frac{v_f - v_i}{a} = \frac{0 - v_0}{a} = -\frac{v_0}{a}$$

and the distance traveled during this braking period is

$$(\Delta x)_2 = \bar{v} t_2 = \left(\frac{v_f + v_i}{2} \right) t_2 = \left(\frac{0 + v_0}{2} \right) \left(-\frac{v_0}{a} \right) = -\frac{v_0^2}{2a}.$$

Thus, the total distance traveled before coming to a stop is

$$s_{\text{stop}} = (\Delta x)_1 + (\Delta x)_2 = \boxed{v_0 \Delta t_r - \frac{v_0^2}{2a}}.$$

- *P2.58** (a) If a car is a distance $s_{\text{stop}} = v_0 \Delta t_r - \frac{v_0^2}{2a}$ (See the solution to Problem 2.57) from the intersection of length s_i when the light turns yellow, the distance the car must travel before the light turns red is

$$\Delta x = s_{\text{stop}} + s_i = v_0 \Delta t_r - \frac{v_0^2}{2a} + s_i.$$

Assume the driver does not accelerate in an attempt to “beat the light” (an extremely dangerous practice!). The time the light should remain yellow is then the time required for the car to travel distance Δx at constant velocity v_0 . This is

$$\Delta t_{\text{light}} = \frac{\Delta x}{v_0} = \frac{v_0 \Delta t_r - \frac{v_0^2}{2a} + s_i}{v_0} = \boxed{\Delta t_r - \frac{v_0}{2a} + \frac{s_i}{v_0}}.$$

- (b) With $s_i = 16 \text{ m}$, $v = 60 \text{ km/h}$, $a = -2.0 \text{ m/s}^2$, and $\Delta t_r = 1.1 \text{ s}$,

$$\Delta t_{\text{light}} = 1.1 \text{ s} - \frac{60 \text{ km/h}}{2(-2.0 \text{ m/s}^2)} \left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}} \right) + \frac{16 \text{ m}}{60 \text{ km/h}} \left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}} \right) = \boxed{6.23 \text{ s}}.$$

- *P2.59** (a) As we see from the graph, from about -50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.

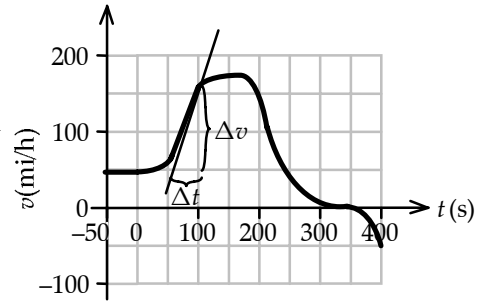


FIG. P2.59(a)

- (b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155 - 45) \text{ mi/h}}{(100 - 50) \text{ s}} = \boxed{2.2 \text{ (mi/h)/s}} = 0.98 \text{ m/s}^2.$$

- (c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

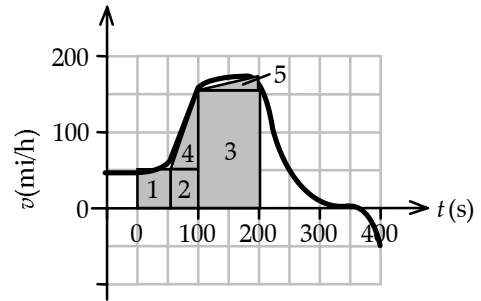


FIG. P2.59(c)

$$\begin{aligned} \Delta x_{0 \rightarrow 200 \text{ s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\ &\approx (50 \text{ mi/h})(50 \text{ s}) + (50 \text{ mi/h})(50 \text{ s}) \\ &\quad + (160 \text{ mi/h})(100 \text{ s}) \\ &\quad + \frac{1}{2}(50 \text{ s})(100 \text{ mi/h}) \\ &\quad + \frac{1}{2}(100 \text{ s})(170 \text{ mi/h} - 160 \text{ mi/h}) \\ &= 24\,000(\text{mi/h})(\text{s}) \end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As $1 \text{ h} = 3\,600 \text{ s}$,

$$\Delta x_{0 \rightarrow 200 \text{ s}} \approx \left(\frac{24\,000 \text{ mi}}{3\,600 \text{ s}} \right) (\text{s}) = \boxed{6.7 \text{ mi}}.$$

***P2.60** Average speed of every point on the train as the first car passes Liz:

$$\frac{\Delta x}{\Delta t} = \frac{8.60 \text{ m}}{1.50 \text{ s}} = 5.73 \text{ m/s}.$$

The train has this as its instantaneous speed halfway through the 1.50 s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60 \text{ m}}{1.10 \text{ s}} = 7.82 \text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50 \text{ s}) + \frac{1}{2}(1.10 \text{ s}) = 1.30 \text{ s}$$

$$\text{so the acceleration is: } a_x = \frac{\Delta v_x}{\Delta t} = \frac{7.82 \text{ m/s} - 5.73 \text{ m/s}}{1.30 \text{ s}} = \boxed{1.60 \text{ m/s}^2}.$$

P2.61 The rate of hair growth is a velocity and the rate of its increase is an acceleration. Then $v_{xi} = 1.04 \text{ mm/d}$ and $a_x = 0.132 \left(\frac{\text{mm/d}}{\text{w}} \right)$. The increase in the length of the hair (i.e., displacement) during a time of $t = 5.00 \text{ w} = 35.0 \text{ d}$ is

$$\begin{aligned} \Delta x &= v_{xi}t + \frac{1}{2}a_x t^2 \\ \Delta x &= (1.04 \text{ mm/d})(35.0 \text{ d}) + \frac{1}{2}(0.132 \text{ mm/d} \cdot \text{w})(35.0 \text{ d})(5.00 \text{ w}) \end{aligned}$$

$$\text{or } \boxed{\Delta x = 48.0 \text{ mm}}.$$

P2.62 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

(0 to 1)	$v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$	so	$v_f = 120 \text{ m/s}$
	$120 = 80.0 + (4.00)t$	giving	$t = 10.0 \text{ s}$

(1 to 2)	$0 - (120)^2 = 2(-9.80)(x_f - x_i)$	giving	$x_f - x_i = 735 \text{ m}$
	$0 - 120 = -9.80t$	giving	$t = 12.2 \text{ s}$
This is the time of maximum height of the rocket.			

(2 to 3)	$v_f^2 - 0 = 2(-9.80)(-1\,735)$		
	$v_f = -184 = (-9.80)t$	giving	$t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

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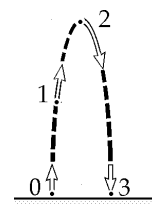


FIG. P2.62

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	-9.80
#3	Fall to Earth	41.0	0	-184	-9.80

P2.63 Distance traveled by motorist = $(15.0 \text{ m/s})t$

Distance traveled by policeman = $\frac{1}{2}(2.00 \text{ m/s}^2)t^2$

(a) intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0 \text{ s}}$

(b) $v(\text{officer}) = (2.00 \text{ m/s}^2)t = \boxed{30.0 \text{ m/s}}$

(c) $x(\text{officer}) = \frac{1}{2}(2.00 \text{ m/s}^2)t^2 = \boxed{225 \text{ m}}$

P2.64 Area A_1 is a rectangle. Thus, $A_1 = hv = v_{xi}t$.

Area A_2 is triangular. Therefore $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}$$

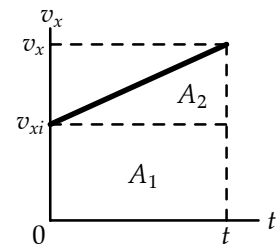


FIG. P2.64

The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

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- P2.65** (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x = \frac{1}{2}(v + v_i)t_1, 100 - x = v(10.2 - t_1) \text{ and } v = v_i + at_1.$$

The first two give

$$100 = \left(10.2 - \frac{1}{2}t_1\right)v = \left(10.2 - \frac{1}{2}t_1\right)at_1$$

$$a = \frac{200}{(20.4 - t_1)t_1}.$$

$$\text{For Maggie: } a = \frac{200}{(18.4)(2.00)} = \boxed{5.43 \text{ m/s}^2}$$

$$\text{For Judy: } a = \frac{200}{(17.4)(3.00)} = \boxed{3.83 \text{ m/s}^2}$$

- (b) $v = a_1t$

$$\text{Maggie: } v = (5.43)(2.00) = \boxed{10.9 \text{ m/s}}$$

$$\text{Judy: } v = (3.83)(3.00) = \boxed{11.5 \text{ m/s}}$$

- (c) At the six-second mark

$$x = \frac{1}{2}at_1^2 + v(6.00 - t_1)$$

$$\text{Maggie: } x = \frac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3 \text{ m}$$

$$\text{Judy: } x = \frac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7 \text{ m}$$

Maggie is ahead by $\boxed{2.62 \text{ m}}$.

P2.66 $a_1 = 0.100 \text{ m/s}^2$

$$x = 1000 \text{ m} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2$$

$$1000 = \frac{1}{2}a_1t_1^2 + a_1t_1\left(-\frac{a_1t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2$$

$$t_2 = \frac{a_1t_1}{-a_2} = \frac{12.9}{0.500} \approx 26 \text{ s}$$

$$a_2 = -0.500 \text{ m/s}^2$$

$$t = t_1 + t_2 \text{ and } v_1 = a_1t_1 = -a_2t_2$$

$$1000 = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$$

$$t_1 = \sqrt{\frac{20000}{1.20}} = \boxed{129 \text{ s}}$$

$$\text{Total time} = t = \boxed{155 \text{ s}}$$

P2.67 Let the ball fall 1.50 m. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$$v_{xf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(-1.50 \text{ m})$$

$$v_{xf} = -5.42 \text{ m/s}$$

and its stopping is described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42 \text{ m/s})^2 + 2a_x(-10^{-2} \text{ m})$$

$$a_x = \frac{-29.4 \text{ m}^2/\text{s}^2}{-2.00 \times 10^{-2} \text{ m}} = +1.47 \times 10^3 \text{ m/s}^2.$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3 \text{ m/s}^2}$.

***P2.68** (a) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. We assume the package starts from rest.

$$-145 \text{ m} = 0 + 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

$$t = \sqrt{\frac{2(-145 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{5.44 \text{ s}}$$

$$(b) \quad x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(5.18 \text{ s})^2 = -131 \text{ m}$$

$$\text{distance fallen} = |x_f| = \boxed{131 \text{ m}}$$

$$(c) \quad \text{speed} = |v_{xf}| = |v_{xi} + a_x t| = |0 + (-9.8 \text{ m/s}^2)5.18 \text{ s}| = \boxed{50.8 \text{ m/s}}$$

(d) The remaining distance is

$$145 \text{ m} - 131.5 \text{ m} = 13.5 \text{ m}.$$

During deceleration,

$$v_{xi} = -50.8 \text{ m/s}, v_{xf} = 0, x_f - x_i = -13.5 \text{ m}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i):$$

$$0 = (-50.8 \text{ m/s})^2 + 2a_x(-13.5 \text{ m})$$

$$a_x = \frac{-2580 \text{ m}^2/\text{s}^2}{2(-13.5 \text{ m})} = +95.3 \text{ m/s}^2 = \boxed{95.3 \text{ m/s}^2 \text{ upward}}.$$

P2.69 (a) $y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2$,
 $4.90t^2 + 2.00t - 50.0 = 0$

$$t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$$t = \boxed{3.00 \text{ s}} \text{ after the first stone is thrown.}$$

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00 \text{ s}$
 substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$:

$$v_{i2} = \boxed{15.3 \text{ m/s}} \text{ downward}$$

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4 \text{ m/s}}$ downward
 $v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8 \text{ m/s}}$ downward

P2.70 (a) $d = \frac{1}{2}(9.80)t_1^2$ $d = 336t_2$
 $t_1 + t_2 = 2.40$ $336t_2 = 4.90(2.40 - t_2)^2$
 $4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$
 $t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765 \text{ s}$ so $d = 336t_2 = \boxed{26.4 \text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2 \text{ m}$, an error of $\boxed{6.82\%}$.

P2.71 (a) In walking a distance Δx , in a time Δt , the length of rope ℓ is only increased by $\Delta x \sin \theta$.
 $\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t} \sin \theta$.

$$v = \frac{\Delta x}{\Delta t} \sin \theta = v_{\text{boy}} \frac{x}{\ell} = \boxed{v_{\text{boy}} \frac{x}{\sqrt{x^2 + h^2}}}$$

(b) $a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{\ell} \frac{dx}{dt} + v_{\text{boy}} x \frac{d}{dt} \left(\frac{1}{\ell} \right)$
 $a = v_{\text{boy}} \frac{v_{\text{boy}}}{\ell} - \frac{v_{\text{boy}} x}{\ell^2} \frac{d\ell}{dt}$, but $\frac{d\ell}{dt} = v = v_{\text{boy}} \frac{x}{\ell}$
 $\therefore a = \frac{v_{\text{boy}}^2}{\ell} \left(1 - \frac{x^2}{\ell^2} \right) = \frac{v_{\text{boy}}^2}{\ell} \frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{(x^2 + h^2)^{3/2}}}$

(c) $\frac{v_{\text{boy}}^2}{h}, 0$

(d) $v_{\text{boy}}, 0$

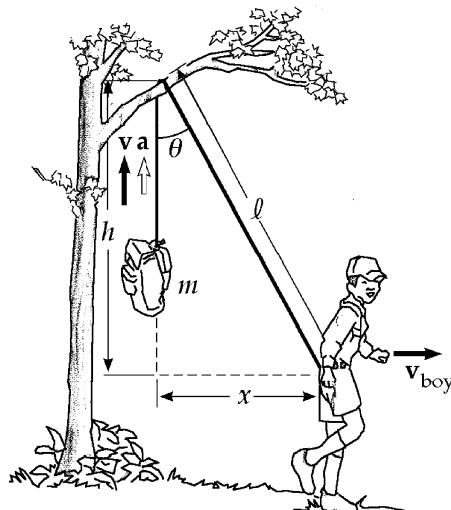


FIG. P2.71

P2.72 $h = 6.00 \text{ m}$, $v_{\text{boy}} = 2.00 \text{ m/s}$ $v = \frac{\Delta x}{\Delta t} \sin \theta = v_{\text{boy}} \frac{x}{\ell} = \frac{v_{\text{boy}} x}{(x^2 + h^2)^{1/2}}.$

However, $x = v_{\text{boy}} t : \therefore v = \frac{v_{\text{boy}}^2 t}{(v_{\text{boy}}^2 t^2 + h^2)^{1/2}} = \frac{4t}{(4t^2 + 36)^{1/2}}.$

(a)

$t(\text{s})$	$v(\text{m/s})$
0	0
0.5	0.32
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71

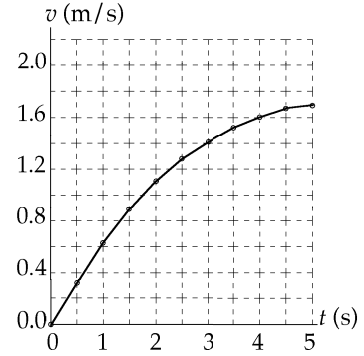


FIG. P2.72(a)

(b) From problem 2.71 above, $a = \frac{h^2 v_{\text{boy}}^2}{(x^2 + h^2)^{3/2}} = \frac{h^2 v_{\text{boy}}^2}{(v_{\text{boy}}^2 t^2 + h^2)^{3/2}} = \frac{144}{(4t^2 + 36)^{3/2}}.$

$t(\text{s})$	$a(\text{m/s}^2)$
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3	0.24
3.5	0.18
4	0.14
4.5	0.11
5	0.09

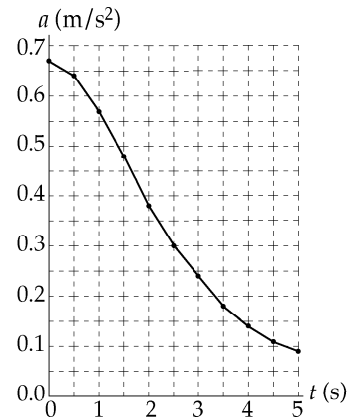


FIG. P2.72(b)

P2.73 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2} (3.50 \text{ m/s}^2) (t_k + 1.00)^2 = \frac{1}{2} (4.90 \text{ m/s}^2) (t_k)^2 = x_k$$

$$t_k + 1.00 = 1.183 t_k$$

$$t_k = \boxed{5.46 \text{ s}}.$$

(b) $x_k = \frac{1}{2} (4.90 \text{ m/s}^2) (5.46 \text{ s})^2 = \boxed{73.0 \text{ m}}$

(c) $v_k = (4.90 \text{ m/s}^2) (5.46 \text{ s}) = \boxed{26.7 \text{ m/s}}$
 $v_s = (3.50 \text{ m/s}^2) (6.46 \text{ s}) = \boxed{22.6 \text{ m/s}}$

P2.74

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpt time t (s)
0.00	5.00				
0.25	5.75	0.75	0.25	3.00	0.13
0.50	6.40	0.65	0.25	2.60	0.38
0.75	6.94	0.54	0.25	2.16	0.63
1.00	7.38	0.44	0.25	1.76	0.88
1.25	7.72	0.34	0.25	1.36	1.13
1.50	7.96	0.24	0.25	0.96	1.38
1.75	8.10	0.14	0.25	0.56	1.63
2.00	8.13	0.03	0.25	0.12	1.88
2.25	8.07	-0.06	0.25	-0.24	2.13
2.50	7.90	-0.17	0.25	-0.68	2.38
2.75	7.62	-0.28	0.25	-1.12	2.63
3.00	7.25	-0.37	0.25	-1.48	2.88
3.25	6.77	-0.48	0.25	-1.92	3.13
3.50	6.20	-0.57	0.25	-2.28	3.38
3.75	5.52	-0.68	0.25	-2.72	3.63
4.00	4.73	-0.79	0.25	-3.16	3.88
4.25	3.85	-0.88	0.25	-3.52	4.13
4.50	2.86	-0.99	0.25	-3.96	4.38
4.75	1.77	-1.09	0.25	-4.36	4.63
5.00	0.58	-1.19	0.25	-4.76	4.88

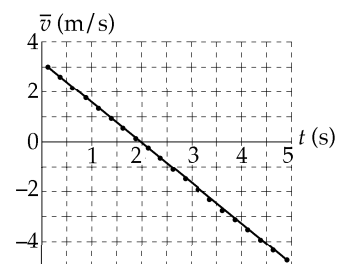


FIG. P2.74

TABLE P2.74

acceleration = slope of line is constant.

$$\bar{a} = -1.63 \text{ m/s}^2 = \boxed{1.63 \text{ m/s}^2 \text{ downward}}$$

- P2.75** The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

Now $\frac{dy}{dt}$ is v_B , the unknown velocity of B ; and $\frac{dx}{dt} = -v$. From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y} \left(\frac{dx}{dt} \right) = -\frac{x}{y} (-v).$$

But $\frac{y}{x} = \tan \alpha$ so $v_B = \left(\frac{1}{\tan \alpha} \right) v$. When $\alpha = 60.0^\circ$, $v_B = \frac{v}{\tan 60.0^\circ} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$.

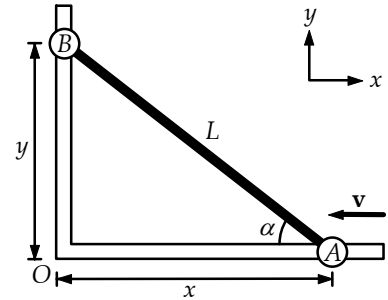


FIG. P2.75

ANSWERS TO EVEN PROBLEMS

- | | | | |
|--------------|---|--------------|---|
| P2.2 | (a) 2×10^{-7} m/s; 1×10^{-6} m/s;
(b) 5×10^8 yr | P2.24 | (a) 1.88 km; (b) 1.46 km;
(c) see the solution;
(d) (i) $x_1 = (1.67 \text{ m/s}^2)t^2$;
(ii) $x_2 = (50 \text{ m/s})t - 375 \text{ m}$;
(iii) $x_3 = (250 \text{ m/s})t - (2.5 \text{ m/s}^2)t^2 - 4375 \text{ m}$;
(e) 37.5 m/s |
| P2.4 | (a) 50.0 m/s; (b) 41.0 m/s | P2.26 | 958 m |
| P2.6 | (a) 27.0 m;
(b) $27.0 \text{ m} + (18.0 \text{ m/s})\Delta t + (3.00 \text{ m/s}^2)(\Delta t)^2$;
(c) 18.0 m/s | P2.28 | (a) $x_f = (30.0t - t^2) \text{ m}$; $v_f = (30.0 - 2t) \text{ m/s}$;
(b) 225 m |
| P2.8 | (a), (b), (c) see the solution; 4.6 m/s^2 ; (d) 0 | P2.30 | $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2$; 3.10 m/s |
| P2.10 | 5.00 m | P2.32 | (a) 35.0 s; (b) 15.7 m/s |
| P2.12 | (a) 20.0 m/s; 5.00 m/s; (b) 262 m | P2.34 | (a) $1.12 \times 10^{11} \text{ m/s}^2$; (b) $4.67 \times 10^{-5} \text{ s}$ |
| P2.14 | (a) see the solution;
(b) 1.60 m/s^2 ; 0.800 m/s^2 | P2.36 | (a) False unless the acceleration is zero;
see the solution; (b) True |
| P2.16 | (a) 13.0 m/s; (b) 10.0 m/s; 16.0 m/s;
(c) 6.00 m/s^2 ; (d) 6.00 m/s^2 | P2.38 | Yes; 212 m; 11.4 s |
| P2.18 | see the solution | P2.40 | (a) -4.90 m; -19.6 m; -44.1 m;
(b) -9.80 m/s; -19.6 m/s; -29.4 m/s |
| P2.20 | (a) 6.61 m/s; (b) -0.448 m/s^2 | P2.42 | 1.79 s |
| P2.22 | (a) $-21.8 \text{ mi/h} \cdot \text{s} = -9.75 \text{ m/s}^2$;
(b) $-22.2 \text{ mi/h} \cdot \text{s} = -9.94 \text{ m/s}^2$;
(c) $-22.8 \text{ mi/h} \cdot \text{s} = -10.2 \text{ m/s}^2$ | | |

54 *Motion in One Dimension*

- P2.44** No; see the solution
- P2.46** The second ball is thrown at speed
 $v_i = \sqrt{gh}$
- P2.48** (a) 510 m; (b) 20.4 s
- P2.50** (a) 96.0 ft/s;
(b) $a = 3.07 \times 10^3 \text{ ft/s}^2$ upward;
(c) $\Delta t = 3.13 \times 10^{-2} \text{ s}$
- P2.52** 38.2 m
- P2.54** (a) and (b) see the solution; (c) -4 m/s^2 ;
(d) 34 m; (e) 28 m
- P2.56** 0.222 s
- P2.58** (a) see the solution; (b) 6.23 s
- P2.60** 1.60 m/s^2
- P2.62** (a) 41.0 s; (b) 1.73 km; (c) -184 m/s
- P2.64** $v_{xi}t + \frac{1}{2}a_xt^2$; displacements agree
- P2.66** 155 s; 129 s
- P2.68** (a) 5.44 s; (b) 131 m; (c) 50.8 m/s;
(d) 95.3 m/s^2 upward
- P2.70** (a) 26.4 m; (b) 6.82%
- P2.72** see the solution
- P2.74** see the solution; $a_x = -1.63 \text{ m/s}^2$